1 Centre of mass

1.1 General centre of mass

Shape $A(x_1, y_1)$ weighted w_1, $B(x_2, y_2)$ weighted w_2, $C(x_3, y_3)$ weighted w_3, centre of mass $G(\overline{x}, \overline{y})$:

$$w_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + w_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + w_3 \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = (w_1 + w_2 + w_3) \begin{pmatrix} \overline{x} \\ \overline{y} \end{pmatrix}$$

1.2 Centre of mass of uniform triangular lamina

$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

1.3 Centre of mass of uniform sector of a circle

Make the origin at the centre of the circle, make x-axis crossing the symmetry of the sector, then

$$distance = \frac{2r \sin \alpha}{3\alpha}$$

$$G\left(0, \frac{2r \sin \alpha}{3\alpha}\right)$$

1.4 Combined centre of mass of two laminae

Shape A, $G_A(x_a, y_a)$ area A_a;
Shape B, $G_B(x_b, y_b)$ area A_b.
Combined centre of mass:

$$A_a \begin{pmatrix} x_a \\ y_a \end{pmatrix} + A_b \begin{pmatrix} x_b \\ y_b \end{pmatrix} = (A_a + A_b) \begin{pmatrix} \overline{x} \\ \overline{y} \end{pmatrix}$$

1.5 Centre of mass of an arc of a circle

Make the origin at the centre of the circle, make x-axis crossing the symmetry of the sector, then

$$distance = \frac{r \sin \alpha}{\alpha}$$

1.6 Centre of mass of a frame

Consider all lines as a dedicated shape, use the general method to calculate its centre then use centre point to represent the shape then apply the general method again.

2 Further Center of Mass

2.1 Center of mass of a lamina

$$\overline{x} = \frac{\int_a^b xy \, dx}{\int_a^b y \, dx}$$

$$\overline{y} = \frac{\int_a^b \frac{1}{2} y^2 \, dx}{\int_a^b y \, dx}$$

Or calculate separately,

$$M = \int_a^b \rho y \, dx$$

$$M\overline{x} = \int_a^b \rho xy \, dx$$

$$M\overline{y} = \int_a^b \frac{1}{2} \rho y^2 \, dx$$

2.2 Center of mass of a solid mass rotated about x-axis

$$\bar{x} = \frac{\int_a^b \pi y^2 x \, dx}{\int_a^b \pi y^2 \, dx}$$

$$M = \int_a^b \rho \pi y^2 \, dx$$

2.3 Center of mass of a solid mass rotated about y-axis

$$\bar{y} = \frac{\int_a^b \pi x^2 y \, dx}{\int_a^b \pi x^2 \, dx}$$

2.4 Center of mass of solid hemisphere

$$d = \frac{3}{8} r$$

2.5 Center of mass of hemisphere shell

$$d = \frac{1}{2} r$$

2.6 Center of mass of circular cone

$$d = \frac{1}{4} h$$

2.7 Center of mass of hollow cone

$$d = \frac{1}{3} h$$

3 Momentum

3.1 Coefficients of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

When $e = 1$, perfect elastic

3.2 Collisions with wall

$$e = \frac{v}{u}$$

4 Stiffness coefficient

4.1 Force by extended string

$$F = \frac{\lambda x}{L}$$

4.2 Energy stored in string

$$W = \frac{\lambda x^2}{2L}$$